Unit 3. Scarcity work and choice

This is a technical unit that introduces you to a number of very important concepts. The concepts will show up again and again in this course and many of your economics courses in the future. Some of the most important concepts are preferences, opportunity cost, indifference curves, income and substitution effects, production functions, and my favorite Latin phrase, that's right, say it with me *ceteris paribus.*

You are introduced for the first time to constrained optimization models that form the foundation for the framework of most economists. Understanding constrained optimization requires knowledge about indifference curves and the feasible set. Finding the optimal equilibrium of a constrained optimization problem is really quite simple, it occurs at the point where the marginal rate of substitution equals the marginal rate of transformation.

Make sure you understand the constrained optimization framework now or suffer later! I am serious!

A Introduction

The context for this unit

A recurring theme thus far has been living standards and how they have evolved over time. Empirically, we have measures which demonstrate a drastic increase in living standards. Last week we introduced isocost lines as a way to model choice of technology.

With the emergence of capitalism and the ascendancy of firms to that of a dominant institution, members of society must seek work from those firms to acquire the material means of well-being. In addition to technology, we now consider a second input into production: labor.

One question we will focus on this unit is: how do individuals decide how much to work? More work equals more money and greater access to the goods and services produced by society, but it also means less free time or leisure as it is referred to by economists.

This unit

The technological revolution that propelled the rise in living standards leaves us with yet another question. Has economic progress yielded an increase in consumption of goods and services, more time to pursue leisure, or both?

This graphic shows the trend in income and working hours. Initially, there is a steep drop in hours worked for all three countries. The negative trend between hours worked and per capita income has continued albeit at a much more modest pace in the European countries. Whereas, in the US income has continued to rise, but hours worked has remained stagnant.

This unit 2

The graphic presented on this slide demonstrates how widely an individual's experience is based on where they are born. An average resident of Mexico has relatively little free time and a lower income than every other country presented, whereas a country like Norway has significantly higher income and much more free time.

To start to make sense of these empirical observations we will introduce a model of decision making. We use a constrained optimization model to gain insight into decision making under the presence of scarcity. Scarcity is an assumption that implies a constraint on our decision; we want more than we are able to have.

The model introduced in this unit will be used repeatedly through the semester.

B. Scarcity and choice

Example grades and study hours

I am confident that all of you are working so hard and studying diligently for this course on a regular basis. Many of you have additional demands on your time with employment in addition to the demands school imposes.

With many of the outcomes we care about, we face the same problem over and over: how do we measure it? The same is true of work. How do we measure work, is one hour studying in the library the equivalent to one hour studying in your favorite pub? Is the environment in which you study the only factor which influences the relationship between study time and grades? When I treat it as such, what assumption am I making? C'mon, say it with me: *ceteris paribus.*

Production function

In the example we develop now, we are seeking to understand how you as a student decide how many hours are devoted to studying. Scarcity enters the problem because more time studying means less time playing. So why study at all, well we assume a positive correlation between the amount of work you put in, measured simply by hours studying, and the grade you receive. That is, more studying equals a higher grade. Work, the number of hours you spend studying, is the input and your grade is the output.

This magical modelling tool known as the production function describes how inputs are converted into outputs. Perhaps describe is not really the most truthful, because it is more of a black box. Holding everything else constant, it is a mathematical statement that says if you use X units of input, then the maximum output you can get is Y.

What can production function tell us

The production function tells us how much output we can expect from a certain amount of input. We are holding everything constant, except the one input we allow to change. In this example it is study time. As you increase study time you are moving along the horizontal axis to the right. At any given amount of studying, say four hours, draw a vertical line up to the production function and a horizontal to the vertical axis to determine what grade that amount of studying would yield.

The marginal product is how much output increases with an additional one unit increase of the input. 4 hours of study yields a mark of 50, 5 hours of study yields a mark of 57, so the additional increase in the grade from a 1 more hour of studying is 57-50 or 7. At 4 hours of studying the marginal product is 7. The marginal product is the slope of the production function.

The production function is increasing, but it increases at a decreasing rate: the production function is concave. As you follow the production function to the right, it goes up, but it becomes flatter the further right you move. This is an important concept: diminishing marginal product. With every additional unit of input the additional output you can expect becomes smaller.

From the production function you can also calculate the average product by finding the slope of the ray from the origin to the point on the production function.

Studying example

The marginal product at one hour of studying is 13, I got this number by subtracting the amount of output from the additional unit on input from the amount of output at the current units of input: a grade of 33 for 2 hours of study minus the grade of 20 for one hour of study yields a marginal product of 13.

If I increase the amount of studying one additional unit, the marginal product tells me by how much my grade would change.

Indifference curves

The production function alone is not sufficient to let us model the decision-making process. The production function simply tells us what is possible, what we need to know in addition is what the agent's preferences are.

The tool we use to represent preferences are indifference curves. The only two things that our decision-making student cares about are grades and free time. Grades are represented on the vertical axis and free time on the horizontal axis. The indifference curve represents all the different combinations of grades and free time that give the same amount of utility to the agent. Like the isocost line, all the combinations of two things that give you a constant something else.

The indifference curves slope downward because we are holding utility constant, more of one good must be offset by less of the other.

While Alexi, the student in our example, is indifferent between points A and D, they prefer to be on the indifference curve through A-D than any of the other two indifference curves presented. The further the indifference curve is from the origin, the higher the utility. Higher indifference curves mean higher utility.

The indifference curves are smooth and convex. Go to grad school if you care about what this means.

Indifference curves are assumed to never cross. This would violate the rule of transitivity.

If A>B and B>C, then by the rule of transitivity A>C. Apply this to indifference curves to understand why they cannot cross.

The further right you move along an indifference curve the flatter it becomes. More on this in a moment.

The slope of the indifference curve is the marginal rate of substitution (MRS). The MRS tells us the rate at which Alexi is willing to accept a lower grade for more free time. It seems reasonable that how much you value free time depends on how much of it you have. If Alexi has a lot of free time, they would be willing to give up more free time for a small increase in their grade than if they had very little free time.

Consider two scenarios: 1) you have very little free time and a very good grade. 2) you have a lot of free time and a really bad grade. In which scenario would you be more willing to sacrifice a reduction in your grade to have an extra hour hanging at the mall with your friends? Probably scenario 1. In scenario 1, your MRS is greater than your MRS in scenario 2; the MRS is the rate at which you are willing to sacrifice a reduction in grade for additional free time.

How does the varying MRS explain the flatter indifference curves? Well the MRS is the slope of the indifference curve. The indifference curve tells the rate at which you would be willing to sacrifice less of one thing to have more of another.

Opportunity cost

The decision about how much to study presents a dilemma, more studying means less free time. The production function we introduced earlier informs us of the opportunity cost of additional free time: more free time means less time studying, less time studying means a lower grade. To get more free time you have to give up the opportunity of a higher grade.

Opportunity costs arise when we must choose between competing ends: doing one action precludes the other.

Opportunity cost example

In this example you have to choose between concert A and concert B. If you go to one concert, you cannot go to the other. These concerts are alternatives and mutually exclusive courses of action. The concept of opportunity cost is linked to the previous concept of economic rent and the associated decision rule. Now the rent considers the opportunity cost of not pursuing the alternative course of action.

The feasible frontier

Here we are introduced to another important element of the constrained optimization problem: the feasible frontier.

Recall that the production function describes how studying is converted to grades. Notice that the axes in this figure are grade on the vertical which is the same as when we looked at the production function. But now, the horizontal axis is free time, not time studying.

The feasible frontier shows us the highest grade that Alexi can achieve given the decision about free time. Anything above the feasible set is not possible. Every combination on or below the feasible frontier constitutes the feasible set.

When we are on the frontier, we are at the highest grade that can be achieved given the choice of free time. The slope of the feasible frontier represents the tradeoff we are constrained to make: if we want more free time we have an opportunity cost of foregone points. The slope of the feasible frontier is the rate at which you can convert free time into points: this is the marginal rate of transformation (MRT).

Earlier, we were introduced to the MRS which represents the tradeoff the decision maker is willing to make. Now we are presented with the MRT which represents the tradeoff the decision maker is constrained to make.

C. decision-making under scarcity

Constrained choice problem

We have now arrived at the final step of modeling the decision-making process. But before we solve for the equilibrium, lets review the key concepts.

Our decision maker Alexi has preferences which we assume are exogenous- preferences are determined outside and independent of what happens in the model. The preferences are represented by the indifference curves. The slope of the indifference curve is the MRS which represents the trade-off Alexi is willing to make between free time and a better grade.

Alexi cannot simply choose to have a lot of free time and a good grade. No, there are some combinations which are not feasible. What is feasible is determined by the production function. The production function allows us to determine the set of feasible combinations. The slope of the frontier of this set is the MRT which gives us the trade-off Alexi is constrained to make: the change in grade that results from a change in free time.

Optimal decision making

Oh yeah, now it is time to find the equilibrium.

Alexi is seeking the combination, and there is only one, which maximizes his utility. The combination which maximizes utility is the combination of free time and grade which resides on the highest possible indifference curve.

Easy, pick one on IC4. It doesn't matter which combination on IC4 since they are indifferent between all of them and it's the highest indifference curve in the image. NO! All of the combinations on IC4 are not feasible given existing technology (recall technology refers to the process by which inputs are converted to outputs- studying into grades in this example).

Point A is combination with the most free time, but B has a better grade. Which does Alexi choose? Neither A nor B. Point C is on higher indifference curve and is thus preferred to either A or B. But the decision maker should never choose a combination inside the feasible set. If you are inside the feasible set, you can do better. From point C, you can have a higher grade without giving up any additional free time or more free time without taking any reduction to the grade by simply moving to the frontier either vertically or horizontally respectively.

The equilibrium occurs at point E. Our equilibrium condition is one where the trade-off the decision-maker is willing to make is equal to the trade-off they are constrained to make.

If MRS does not equal MRT you can do better. Consider point B, the slope of the indifference curve, MRS, is steeper or greater than the slope of the frontier, MRT. Interpret this as the trade-off Alexi is willing to make for an additional hour of free time is greater than the opportunity cost of doing so. You would earn a rent from pursuing more leisure, and if a course of action yields a rent, do it. At point A, the converse is true.

The equilibrium is the point where MRS=MRT and graphically, this is where the indifference curve is tangent to the frontier.

Another example

The state of technology is such that people, on average, could work far less than they currently do and maintain a relatively high standard of living. In fact, this was a key theme in Keynes book from the 1930's titled *Economic possibilities for our grandchildren*.

Improvements in technology over the last few centuries has made this scenario possible. But how then do we incorporate changing technology into our model of decision-making?

A change in technology will shift the production function up or down. An improvement in technology would shift it up. For any given level of units of input the output will be greater. And, at every unit of input, the marginal product is also greater: the slope of the production function is steeper.

Shifting the production function up expands the feasible set, specifically we see that the new technology shifts the vertical intercept of the frontier up.

Optimal decision making

Now to find the equilibrium we consider the preferences of the decision maker Angela in the farming example from the textbook.

The improvement in technology allows her to have more free time and more grain. The new equilibrium where MRS=MRT does in fact provide her more of both, it raises her standard of living and allows her greater utility.

In this example, what did we hold constant: how did we use the *ceteris paribus* assumption? One very important feature that we held constant were her preferences. Her preferences and the willingness to substitute one for the other did not change.

When we shifted the production function up, her marginal product increased which means the opportunity cost of free time is greater. Every hour of free time cost her more grain which is an incentive to work, but at the same time every hour of work yields her more grain which means she wouldn't have to work as much to maintain her living standard. So, does she work more or less? Well…it depends on her preferences, but these two effects are working in different directions.

Next, we turn to an example to help us disentangle these opposing forces.

D. Income and substitution effects

Example working hours

This example introduces you to the budget constraint. This is the feasible set when we are discussing the consumption decision. The budget constraint is what combination of goods and services can be afforded.

The maximum amount you can afford to purchase is your wage, w, multiplied by the number of hours you work, 24-t, where t is free time.

It is the same exact process as before. We have introduced the constraint; we then introduce the decision-makers preferences, which gives us the willingness to substitute one for the other. Now all that remains is to find the combination of consumption and free time that balances the trade-off the decision-maker is willing to make (MRS- the amount of consumption exchanged for an hour of free time) with the trade-off they are constrained to make (MRT- the amount of consumption gained from giving up an hour of free-time which is equal to the wage).

Two important effects

The modelling process will be the same all semester, we decide what variables matter for answering the question at hand, describe the rules by which variables interact, then solve for the equilibrium. Once we have equilibrium, introduce change and study what happens.

In this example, we increase the wage which when holding hours worked constant, increases earnings, but it also increases what you give up for an additional hour of free time. These are the income and substitution effects.

Income effect

Let's assume the decision maker receives an extra $50 of income every period. The additional cash doesn't change the rate at which the decision maker is constrained to make between consumption and free time; the rate at which increasing free time reduces consumption is determined by the wage alone in this example.

The extra cash shifts the budget constraint up. The higher budget constraint leads to a new equilibrium. However, the decision maker did not simply spend the entire $50, they also choose

to increase their free time. Note that this result is wholly contingent upon the decision-makers preferences; it is conceivable that someone would simply spend all the extra cash.

The effect that this unearned income had on the choice of free time is the income effect, which we assume for most goods is always positive or zero.

We hold the opportunity costs constant and then identify how the optimal choice changes with a change in income.

Substitution effect

Assume that instead of unearned income, the wage has increased from $15 to $24. The increased wage shifts the budget constraint up, but the horizontal intercept stays the same. Regardless of the wage, if you don't work, you can't consume. The budget constraint is now steeper; the MRT has increased. The opportunity cost of free time is now greater which incentives the decision maker to work more.

Overall effect

When we assumed additional unearned income, the decision-maker worked fewer hours; however, with an increase in the wage, the decision-maker worked more hours.

Let us dig in a bit further, shall we?

The income effect, when we assumed some unearned income, allowed you to have more consumption for each level of free time. In this situation your MRS is higher: your willingness to sacrifice consumption for free time is greater. In the second scenario considering the substitution effect, the wage increased which caused the budget line to become steeper and thus raised the MRT: the opportunity cost of free time increased which caused you to work more.

In the initial scenario bundle, A was ideal which was found by the tangency of IC2 and the budget constraint.

We then assumed an exogenous increase in unearned income which shifted the budget constraint, visible as the dashed line on the image. The new optimal bundle occurred at C at the tangency with IC4.

Then we assumed a wage increase, the budget constrain becomes steeper. Bundle D is now optimal.

The income effect is the movement in free time A to C and the substitution effect is the movement form C to D.

The overall effect of a wage rise on the amount of free time elected depends on whether the income or substitution effect is greater. In this example the income effect dominated. The substitution effect will always, by assumption, be negative and the income effect positive.

E. Application to technological change

Working hours difference over time.

A recurring theme so far has been rising living standards through time. Here we return to that theme here. In the US, working time has declined over the last century, particularly since the industrial revolution and the abolition of slavery, both occurring towards the end of the 19th century.

Here we have plotted the average estimated free time in 1900 and 2013. The budget constraint is calculated from the real wages observable in historical data. Make the huge assumption that workers can choose how long to work, and we can estimate their indifference curves.

The real wage has increased which causes opposing effects. The income effect is the movement from A to C modeled with the parallel shift up of the budget constraint. However, the rising real wage causes the budget constraint to become steeper, increasing the opportunity cost of free time and causing the movement from C to D. The movement from C to D is the substitution effect.

Working hours cross country differences

When we modeled the change in working hours of American through time, we made use of the *ceteris paribus* assumption.

There are many, many other factors for evolving working hours beyond the purely economic explanation of income and substitution effects. This is perhaps even more important when making cross country comparisons.

How might a worker in Mexico be different from a worker in South Korea or the United States?

Is this a good model

At the conclusion of every unit you should be asking yourself this very question. Get in the habit of thinking critically about the information you are presented with, whether it be in this class or on fox news or CNN.